"Transform Obstacles into Opportunities"

MCCS Middle and High Summer Packet

NAME: ______________________________ **Subject:** Algebra 1

Directions:

- Please complete this packet over the summer.
- It will be collected by your Math teacher during the first day of school, on August 20th, 2018.
- You will be given a GRADE based on the work you complete over the summer.
- All work must be shown to receive credit.
- After reviewing the summer packet in class, a QUIZ will be administered on each of the essential topics.

Name ______________________________

Writing Algebraic Expressions

In **algebraic expressions,** letters such as *x* and *y* are called variables. A variable is used to represent an unspecified number or value.

Practice: Write an algebraic expression for each verbal expression.

1. Four times a number decreased by twelve ________________
2. Three more than the product of five and a number ______________
3. The quotient of two more than a number and eight______________
4. Seven less than twice a number ______________________

Order of Operations:

To evaluate numerical expressions containing more than one operation, use the rules for order of operations. The rules are often summarized using the expression **PEMDAS**

Examples:

Parentheses (Grouping Symbols)	$[(7-4)^2+3]+15$	$\frac{(9-7)^2+6}{11-6}$
Exponents	$=[3^2+3]+15$	$=\frac{2^2+6}{5}$
Multiply or Divide, from left to right	$=[9+3]+15$	$=\frac{4+6}{5}$
Add or Subtract, from left to right	$=12+15$	$=\frac{10}{2}$
		$=5$

Practice: Evaluate each expression.

1. $250 \div [5(3 \bullet 7 + 4)]$

2. $\dfrac{5^2 \cdot 4 - 5 \cdot 4^2}{5(4)}$

3. $\dfrac{1}{2} \cdot 26 - 3^2$

4. $8^2 \div (2 \bullet 8) + 2$

5. $5 + \left[30 - (6-1)^2\right]$

6. $\dfrac{2 \cdot 4^2 - 8 \div 2}{(5+2) \cdot 2}$

Evaluating Algebraic Expressions:

To evaluate algebraic expressions, first replace the variables with their values. Then, use order of operations to calculate the value of the resulting numerical expression.

Example: Evaluate $x^2 - 5(x - y)$ if $x = 6$ and $y = 2$

$$\begin{aligned} x^2 - 5(x - y) &= (6)^2 - 5(6 - 2) \\ &= (6)^2 - 5(4) \\ &= 36 - 5(4) \\ &= 36 - 20 \\ &= 16 \end{aligned}$$

Practice: Evaluate each expression.

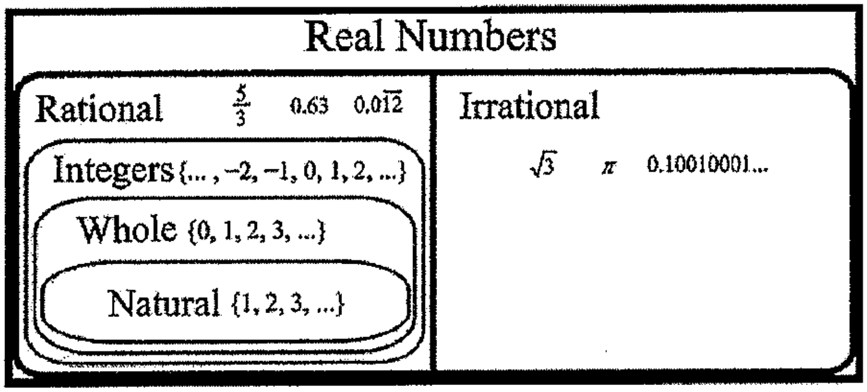
1. $5x^2 - y$ when $x = 4$ and $y = 24$

2. $\dfrac{3xy - 4}{7x}$ when $x = 2$ and $y = 3$

3. $(z + x)^2 + \dfrac{4}{5}x$ when $x = 2$ and $z = 4$

4. $\dfrac{y^2 - 2z^2}{x + y - z}$ when $x = 12, y = 9$, and $z = 4$

The Real Number System:



The **Real** number system is made up of two main sub-groups **Rational numbers** and **Irrational numbers.**

The set of rational numbers includes several subsets: **natural numbers, whole numbers, and integers.**

- **Real Numbers-** any number that can be represented on a number-line.
  - **Rational Numbers-** a number that can be written as the ratio of two integers (this includes decimals that have a definite end or repeating pattern)
    Examples: $2, -5, \frac{-3}{2}, \frac{1}{3}, 0.253, 0.\overline{3}$
    - **Integers-** positive and negative whole numbers and 0
      Examples: $-5, -3, 0, 8$ ...
    - **Whole Numbers** – the counting numbers from 0 to infinity
      Examples: { 0, 1, 2, 3, 4, ...}
    - **Natural Numbers-** the counting numbers from 1 to infinity
      Examples: { 1, 2, 3, 4...}
  - **Irrational Numbers-** Non-terminating, non-repeating decimals (including π, and the square root of any number that is not a perfect square.)
    Examples: 2π, $\sqrt{3}$, $\sqrt{23}$, 3.21211211121111....

Practice: Name all the sets to which each number belongs.

1. -4.2 __________________________
2. $3\sqrt{5}$ __________________________
3. $\frac{5}{3}$ __________________________
4. 9 __________________________
5. $\sqrt{16}$ __________________________
6. $-\frac{8}{2}$ __________________________

Properties of Real Numbers:

Following are properties of Real Numbers that are useful in evaluating and solving algebraic expressions.

Additive Identity	For any number a, $a + 0 = a$.
Multiplicative Identity	For any number a, $a \cdot 1 = a$.
Multiplicative Property of 0	For any number a, $a \cdot 0 = 0$.
Multiplicative Inverse Property	For every number $\frac{a}{b}$, $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property	For any number a, $a = a$.
Symmetric Property	For any numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a may be replaced by b in any expression.

Commutative Properties	For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
Associative Properties	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Additive Inverse Property: For any number a, a + -a = 0

Practice: Name the property illustrated in each equation.

1. $3 \bullet x = x \bullet 3$ ______________________
2. $3a + 0 = 3a$ ______________________
3. $2r + (3r + 4r) = (2r + 3r) + 4r$ ______________________
4. $5y \bullet \frac{1}{5y} = 1$ ______________________
5. $9a + (-9a) = 0$ ______________________
6. $(10b + 12b) + 7b = (12b + 10b) + 7b$ ______________________
7. $5x + 2 = 5x + 2$ ______________________
8. If $9 + 4 = 13$ and $13 = 2 + 11$ then $9 + 4 = 2 + 11$ ______________________
9. If $x = 7$ then $7 = x$ ______________________
10. $3 \bullet 1 = 3$ ______________________

The Distributive Property:

The Distributive Property states for any number a, b, and c:

1. $a(b+c) = ab + ac$ or $(b+c)a = ba + ca$
2. $a(b-c) = ab - ac$ or $(b-c)a = ba - ca$

$4(5a+7)$
$= 4 \cdot 5a + 4 \cdot 7$
$= 20a + 28$

Practice: Simplify **completely** each expression using the distributive property.

1. $7(h - 3)$

2. $-3(2x+5)$

3. $(5x - 9)4$

4. $\frac{1}{2}(14 - 6y)$

5. $3(7x^2 - 3x + 2)$

6. $\frac{1}{4}(16x - 12y + 4z)$

7. $(9 - 2x + 3xy)(-4)$

8. $0.3(40a + 10b - 5)$

Combining Like-Terms:

Terms in algebra are numbers, variables or the product of numbers and variables. In algebraic expressions terms are separated by addition (+) or subtraction (-) symbols. Terms can be combined using addition and subtraction if they are **like-terms.**

Like-terms have the same variables to the same power.

Example of like-terms: $5x^2$ and $-6x^2$

Example of terms that are **NOT** like-terms: $9x^2$ and $15x$

Although both terms have the variable x, they are not being raised to the same power

To combine like-terms using addition and subtraction, add or subtract the numerical factor

Example: Simplify the expression by combining like-terms

$$8x^2+9x-12x+7x^2 = (8+7)x^2+(9-12)x$$
$$= 15x^2+-3x$$
$$= 15x^2-3x$$

Practice: Simplify each expression

1. $5x-9x+2$

2. $3q^2+q-q^2$

3. $c^2+4d^2-7d^2$

4. $5x^2+6x-12x^2-9x+2$

5. $2(3x-4y)+5(x+3y)$

6. $10xy-4(xy+2x^2y)$

Solving Equations with Variables on One-Side:

To solve an equation means to ***find the value*** of the variable. We solve equations by isolating the variable using opposite operations.

Opposite Operations:
Addition (+) & Subtraction (–)
Multiplication (x) & Division (÷)

Please remember...
to do the same step on each side of the equation.

Always check your work by substitution!

Example:

Solve.

$3x - 2 = 10$	
$+2 \quad +2$	Isolate 3x by adding 2 to each side.
$\frac{3x}{3} = \frac{12}{3}$	Simplify Isolate x by dividing each side by 3.
$x = 4$	Simplify

Check your answer.

$3(4) - 2 = 10$	Substitute the value in for the variable.
$12 - 2 = 10$	Simplify
$10 = 10$	Is the equation true? If yes, you solved it correctly!

Practice: Solve each equation.

1. $98 = b + 34$

2. $-14 + y = -2$

3. $8k = -64$

4. $\frac{2}{5}x = 6$

5. $14n - 8 = 34$

6. $8 + \frac{n}{12} = 13$

7. $\frac{3k - 7}{5} = 16$

8. $-\frac{d}{6} + 12 = -7$

Solving Equations with Variables on Each Side:

To solve an equation with the same variable on each side, write an equivalent equation that has the variable on just one side of the equation. Then solve.

Example **Solve $4(2a - 1) = -10(a - 5)$.**

$4(2a - 1) = -10(a - 5)$	Original equation
$8a - 4 = -10a + 50$	Distributive Property
$8a - 4 + 10a = -10a + 50 + 10a$	Add 10a to each side.
$18a - 4 = 50$	Simplify.
$18a - 4 + 4 = 50 + 4$	Add 4 to each side.
$18a = 54$	Simplify.
$\frac{18a}{18} = \frac{54}{18}$	Divide each side by 18.
$a = 3$	Simplify.

The solution is 3.

Practice: Solve each equation.

1. $5+3r=5r-19$

2. $8x+12=4(3+2x)$

3. $-5x-10=2-(x+4)$

4. $6(-3m+1)=5(-2m-2)$

5. $3(d-8)-5=9(d+2)+1$

Solving and Graphing Multi-Step Inequalities:

Note: Solve a multi-step inequality just like you would solve a multi-step equation. However, if you multiply or divide both sides of an inequality by a negative number, then the inequality sign reverses.

Ex.

$2x + 5 > 7$

$\quad -5 \quad -5$

$\frac{2x}{2} > \frac{2}{2}$

$\boxed{x > 1}$

Ex.

$10 \leq -2(x - 4)$

$10 \leq -2x + 8$

$-8 \qquad -8$

$\frac{10}{-2} \leq \frac{-2x}{-2}$

$\boxed{-5 \geq x \text{ or } x \leq -5}$

Find and graph the solution set of each inequality.

1. $3x + 8 > 17$

2. $-6y + 3 > 9 - 7y$

3. $2v + 7 \geq 11$

4. $7 > 3 + \frac{b}{3}$

5. $\frac{c-2}{3} \leq 4$

6. $4b + 4 < 4(5 - 3b)$

7. $2z - 5 < -21 - 2z$

8. $8b - 10 \geq 6(3 - b)$

9. $3x - 5 > 6x + 13$

10. $7(y + 5) - 10 \leq 2y$

Ratios and Proportions:

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes**. They are the first and last terms of the proportion. 5 and 10 are called **means**. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions	For any numbers a, b, c, and d, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 1:

$$\frac{x}{5} = \frac{10}{13}$$

$$x \cdot 13 = 5 \cdot 10$$

$$13x = 50$$

$$\frac{13x}{13} = \frac{50}{13}$$

$$x = \frac{50}{13}$$

Example 2:

$$\frac{x+1}{4} = \frac{3}{4}$$

$$4(x+1) = 3 \cdot 4$$

$$4x + 4 = 12$$
$$-4 \quad -4$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

Practice: Solve each proportion.

1. $\frac{x}{21} = \frac{3}{63}$

2. $\frac{-3}{x} = \frac{2}{8}$

3. $\frac{0.1}{2} = \frac{0.5}{x}$

4. $\frac{9}{y+1} = \frac{18}{54}$

5. $\frac{a-8}{12} = \frac{15}{3}$

6. $\frac{3+y}{4} = \frac{-y}{8}$

Solving for a Specific Variable:

XII. Solving For a Specific Variable

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ. If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 Solve $2x - 4y = 8$ for y.

$$2x - 4y = 8$$
$$2x - 4y - 2x = 8 - 2x$$
$$-4y = 8 - 2x$$
$$\frac{-4y}{-4} = \frac{8 - 2x}{-4}$$
$$y = \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4}$$

The value of y is $\frac{2x - 8}{4}$.

Example 2 Solve $3m - n = km - 8$ for m.

$$3m - n = km - 8$$
$$3m - n - km = km - 8 - km$$
$$3m - n - km = -8$$
$$3m - n - km + n = -8 + n$$
$$3m - km = -8 + n$$
$$m(3 - k) = -8 + n$$
$$\frac{m(3 - k)}{3 - k} = \frac{-8 + n}{3 - k}$$
$$m = \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k}$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Practice: Solve each equation or formula for the variable specified.

1. $15x + 1 = y$ for x

2. $x(4 - k) = p$ for k

3. $7x + 3y = m$ for y

4. $P = 2l + 2w$ for w

Rate of Change and Slope:

Find Slope

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line

Example 1 Find the slope of the line that passes through (−3, 5) and (4, −2).

Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-2 - 5}{4 - (-3)}$ $y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3$

$= \frac{-7}{7}$ Simplify.

$= -1$

Example 2 Find the value of r so that the line through $(10, r)$ and $(3, 4)$ has a slope of $-\frac{2}{7}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$-\frac{2}{7} = \frac{4 - r}{3 - 10}$ $m = -\frac{2}{7}, y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10$

$-\frac{2}{7} = \frac{4 - r}{-7}$ Simplify.

$-2(-7) = 7(4 - r)$ Cross multiply.

$14 = 28 - 7r$ Distributive Property

$-14 = -7r$ Subtract 28 from each side.

$2 = r$ Divide each side by −7.

Practice:

Find the slope of the line that passes through each pair of points.

1. $(4,9), (1,-6)$
3. $(4, 3.5), (-4, 3.5)$
2. $(2,5), (6,2)$
4. $(1, -2), (-2, -5)$

Determine the value of r so the line that passes through ach pair of points has the given slope.

5. $(6,8), (r, -2), m = 1$
6. $(10, r), (3,4), m = -\frac{2}{7}$

Linear Equations in Slope-Intercept Form:

Slope-Intercept Form

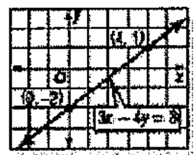
Slope-Intercept Form	$y = mx + b$, where m is the given slope and b is the y-intercept

Example 1 **Write an equation of the line whose slope is −4 and whose y-intercept is 3.**

$y = mx + b$ Slope-intercept form

$y = -4x + 3$ Replace m with −4 and b with 3.

Example 2 Graph $3x - 4y = 8$.



$3x - 4y = 8$ Original equation

$-4y = -3x + 8$ Subtract $3x$ from each side.

$\frac{-4y}{-4} = \frac{-3x + 8}{-4}$ Divide each side by −4.

$y = \frac{3}{4}x - 2$ Simplify.

The y-intercept of $y = \frac{3}{4}x - 2$ is −2 and the slope is $\frac{3}{4}$. So graph the point (0, −2). From this point, move up 3 units and right 4 units. Draw a line passing through both points.

Practice:

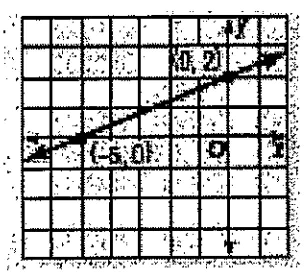
Write an equation of the line with the given slop and y-intercept.

1. slope: $\frac{1}{4}$, y-intercept: 3

2. slope: −2.5, y-intercept: 3.5

Write an equation of the line shown in each graph.

3.



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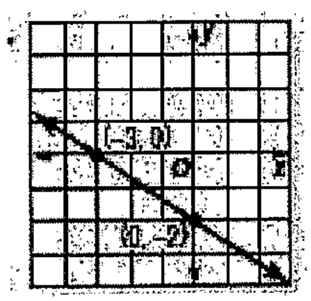
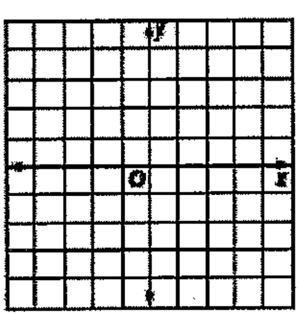
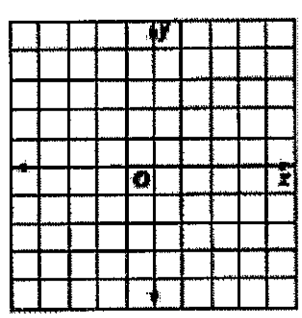


Graph each equation.

5. $y = -\frac{1}{2}x + 2$



6. $6x + 3y = 6$



Solving Word Problems:

Translate each word problem into an algebraic equation, using x for the unknown, and solve. Write a "**let x =**" for each unknown; write an equation; solve the equation; substitute the value for x into the let statements(s) to answer the question.

For Example:

Kara is going to Maui on vacation. She paid $325 for her plane ticket and is spending $125 each night for the hotel. How many nights can she stay in Maui if she has $1200?

Step 1: What are you asked to fine? Let variables represent what you are asked to find.

How many nights can Kara stay in Maui?

Let x = The number of nights Kara can stay in Maui

Step 2: Write an equation to represent the relationship in the problem.

$325 + 125x = 1200$

Step 3: Solve the equation for the unknown

$$\begin{aligned} 325 + 125x &= 1200 \\ -325 \qquad &\quad -325 \\ 125x &= 875 \\ x &= 7 \end{aligned}$$

Kara can spend 7 nights in Maui

Practice: Write an algebraic equation to model each situation. Then solve the equation and answer the question.

1. A video store charges a one-time membership fee of $12.00 plus $1.50 per video rental. How many videos did Stewart rent if he spends $72.00?

2. Darel went to the mall and spent $41. He bought several t-shirts that each cost $12 and he bought 1 pair of socks for $5. How many t-shirts did Darel buy?

3. Nick is 30 years less than 3 times Ray's age. If the sum of their ages is 74, how old are each of the men?

4. Three-fourths of the student body attended the pep-rally. If there were 1230 students at the pep rally, how many students are there in all?

5. Sarah drove 3 hours more than Michael on their trip to Texas. If the trip took 37 hours, how long did Sarah and Michael each drive?

6. Bicycle city makes custom bicycles. They charge $160 plus $80 for each day that it takes to build the bicycle. If you have $480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?